



# Compositional Schedulability Analysis

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PRECISE (Penn Research in Embedded Computing  
and Integrated Systems Engineering) Center

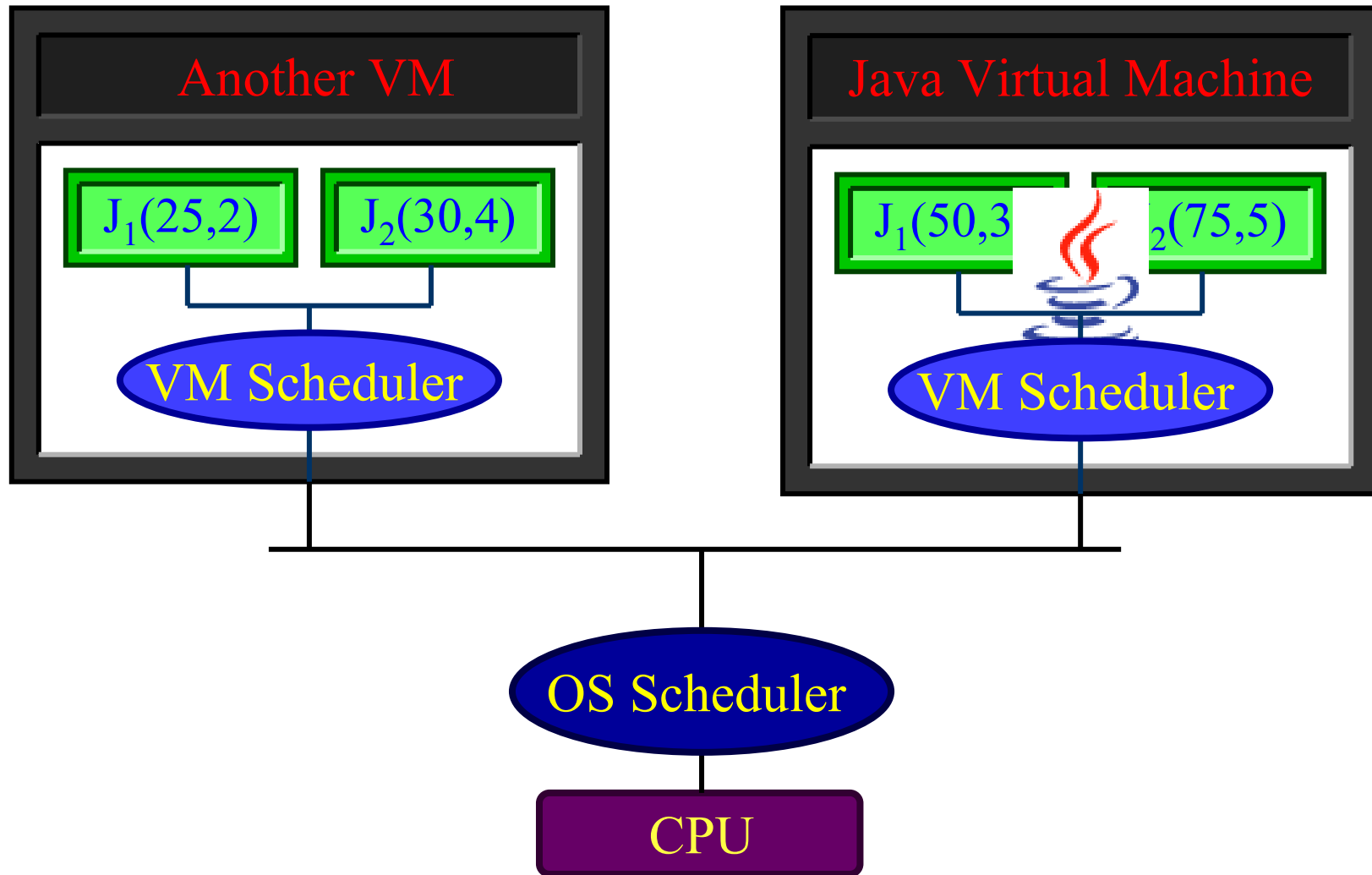
Department of Computer and Information Science  
University of Pennsylvania

*October 19, 2008*

*Workshop on Foundations and Applications of Component-based  
Design (WFCD'2008)*

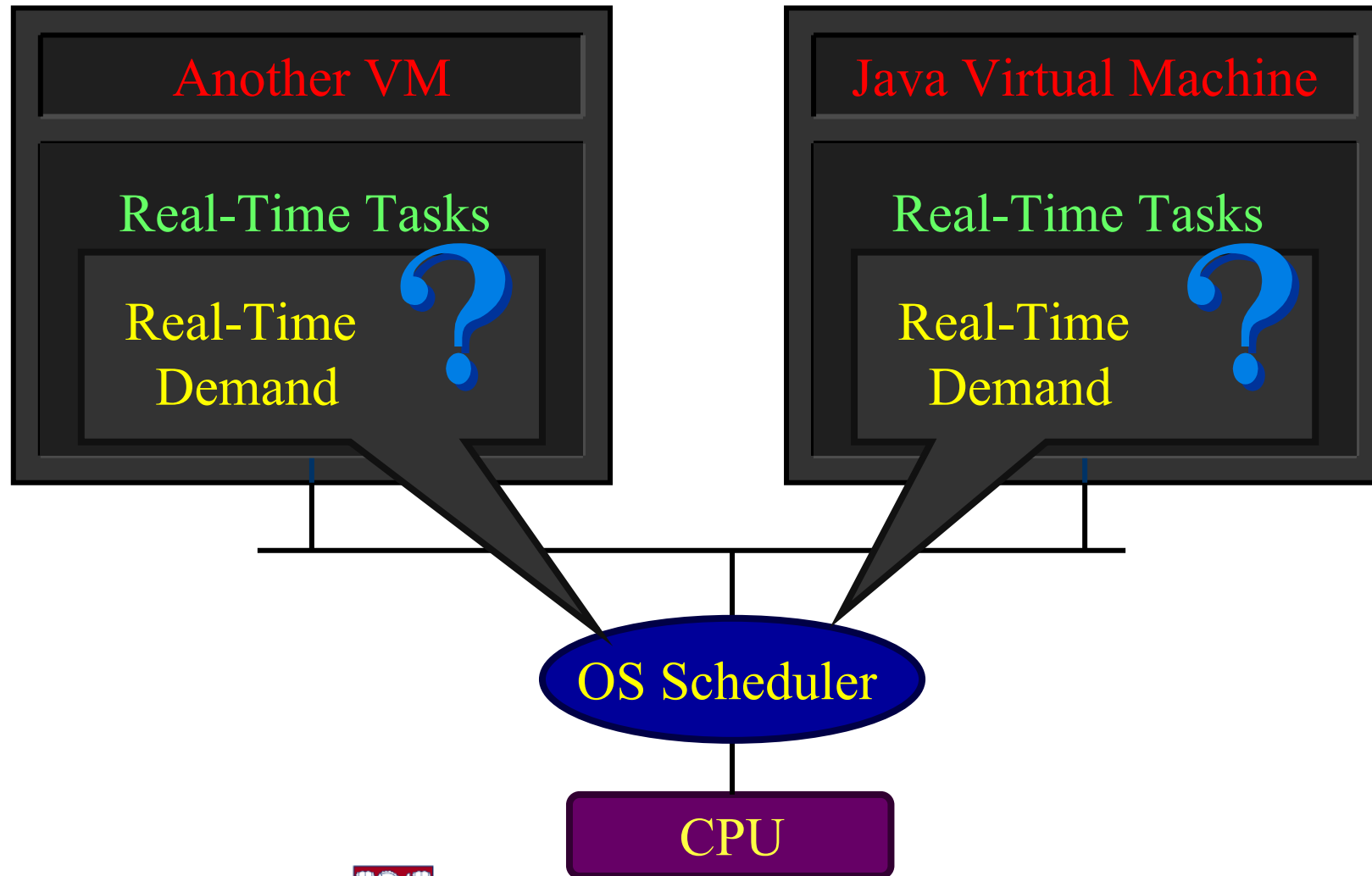
# Motivating Example

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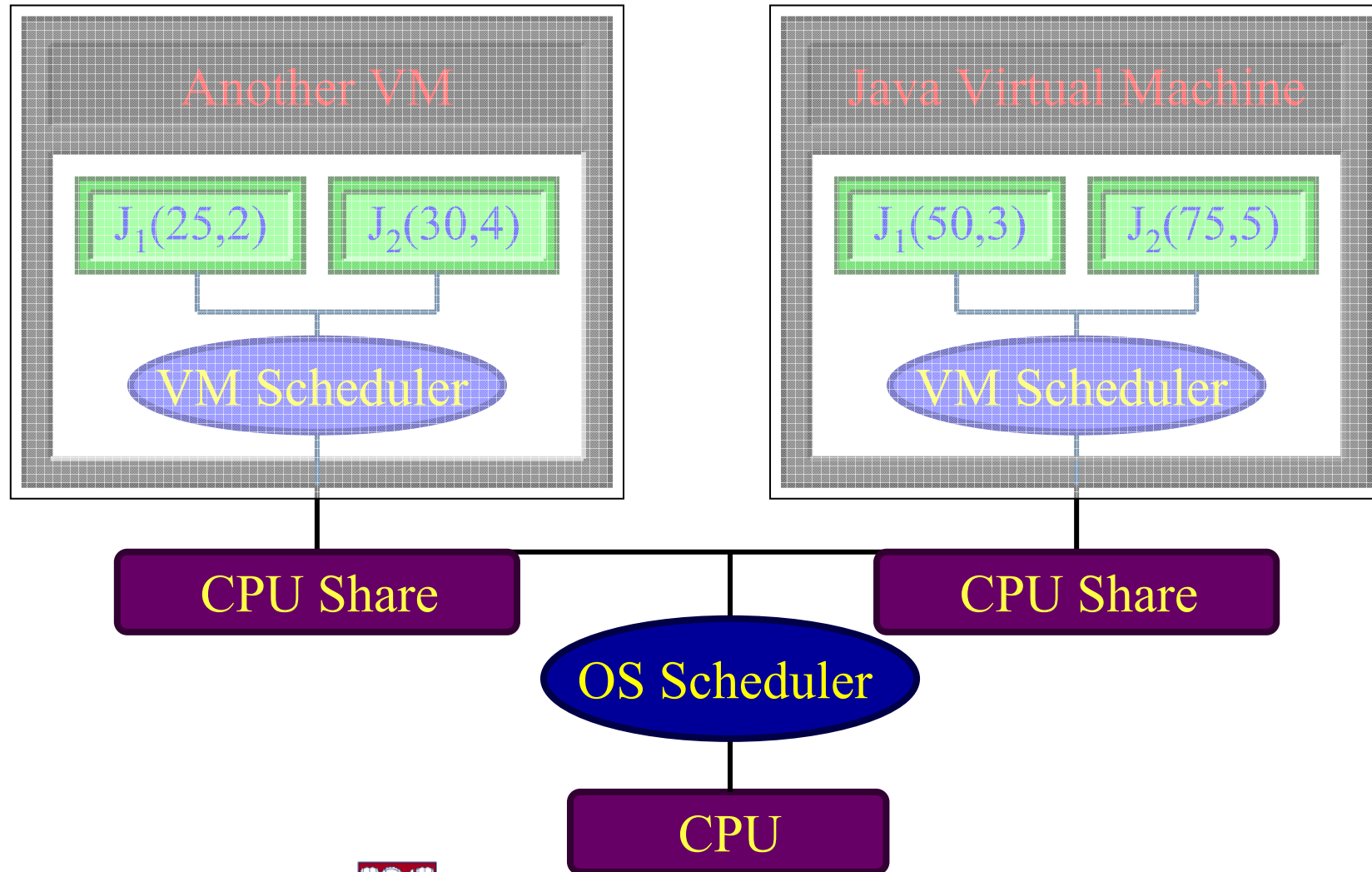


# Component Abstraction

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# Component Abstraction



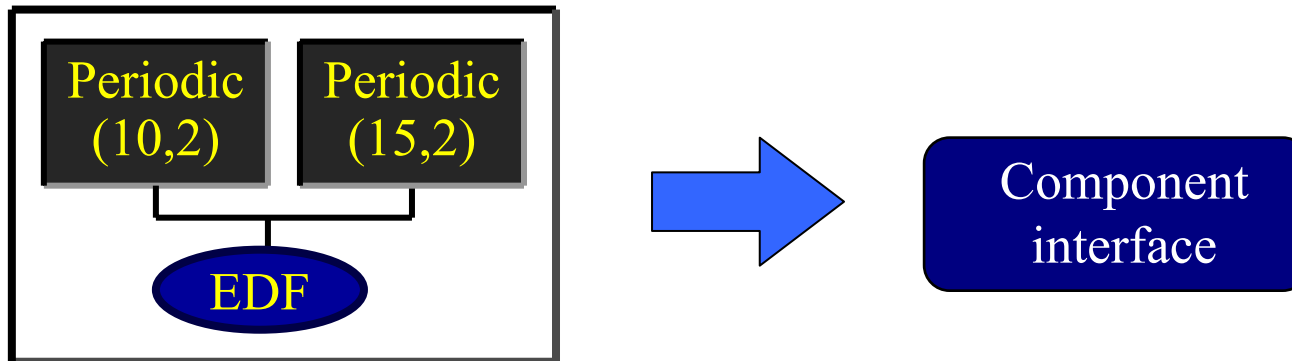


# The CSA Problem Statement

# Two Problems: Abstraction & Composition

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- *Abstraction Problem*: abstracts the real-time requirements of component (application) with *interface*

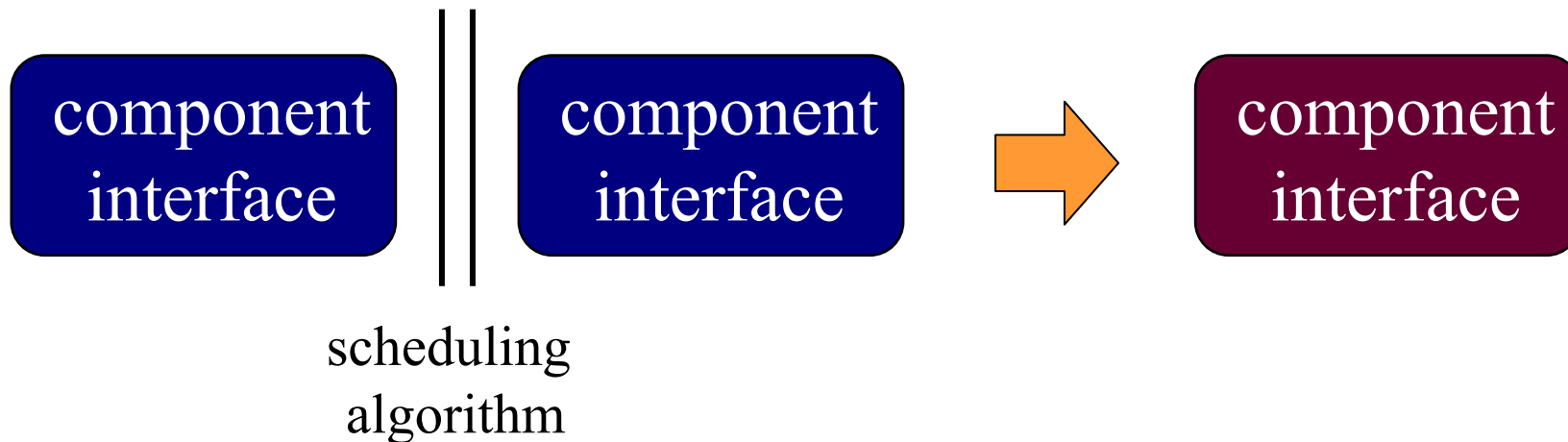


- Compute the minimum real-time requirements necessary for guaranteeing the schedulability of a component

# Two Problems: Abstraction & Composition

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- *Composition Problem*: composes component-level properties into system-level (or next-level component) properties



# Compositionality

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- Compositionality:
  - system-level properties can be established by composing independently analyzed component-level properties
- Compositional reasoning based on assume/guarantee paradigm
  - components are combined to form a system such that properties established at the component-level still hold at the system level.
- Compositional schedulability analysis using the demand/supply bounds
  - Establish the system-level timing properties by combining component-level timing properties through interfaces





# Resource Satisfiability Analysis

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- Given a task set and a resource model, resource satisfiability analysis is to determine if, for every time,

resource demand,  
which a task set needs  
under  
a scheduling algorithm

$\leq$

(minimum possible)  
resource supply

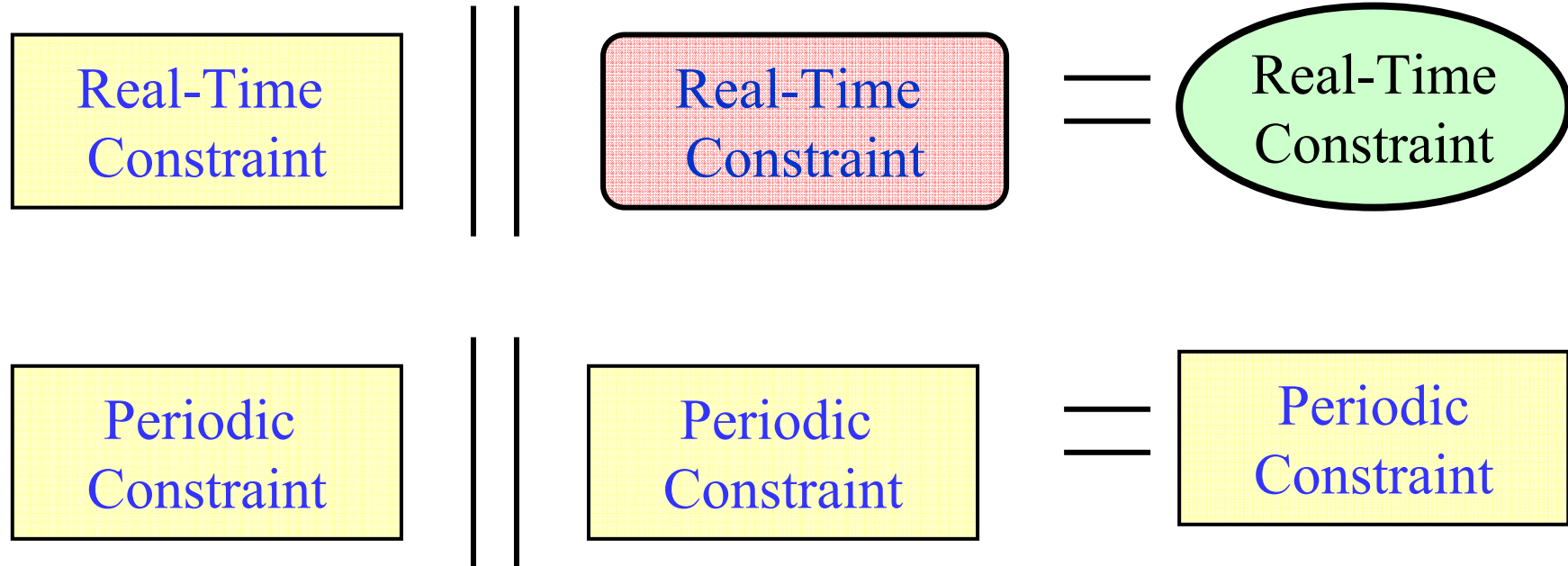


# Resource Demand Models

# Real-time demand composition

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- Combine real-time requirements of multiple tasks into real-time requirement of a single task



EDF / RM

# Non-composable periodic models?

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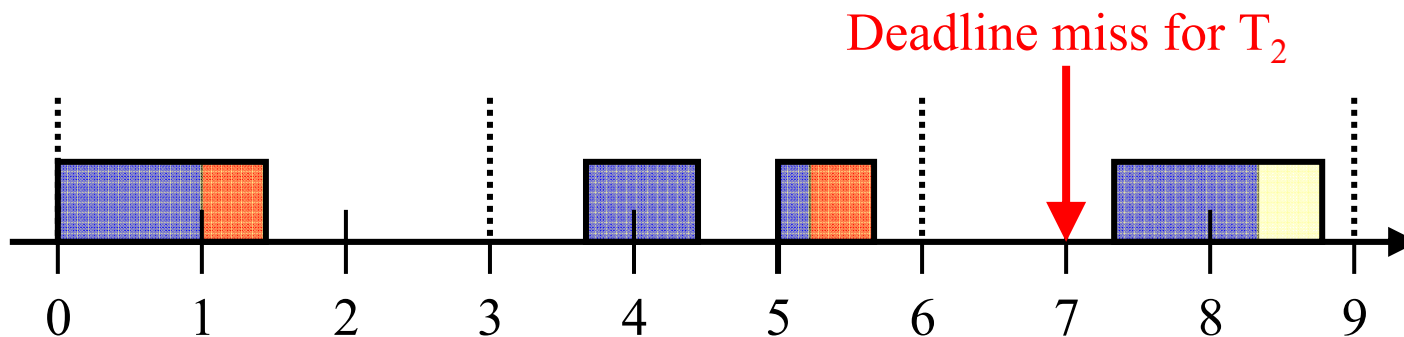
- What are right abstraction levels for real-time components?
- (execution time, period)
- $P1 = (p1, e1)$ ; e.g., (3,1)
- $P2 = (p2, e2)$ ; e.g., (7,1)
- What is  $P1 \parallel P2$ ?
  - $(LCM(p1, p2), e1 * n1 + e2 * n2)$ ; e.g., (21,10)  
where  $n1 * p1 = n2 * p2 = LCM(p1, p2)$
- What is the problem?
  - $beh(P1) \parallel beh(P2) = beh(P1 \parallel P2)$ ?
- Can we do
  - $(P1 \parallel P2) \parallel P3 = P \parallel P3$ , where  $P = P1 \parallel P2$ ?



# Simple Observation (1)

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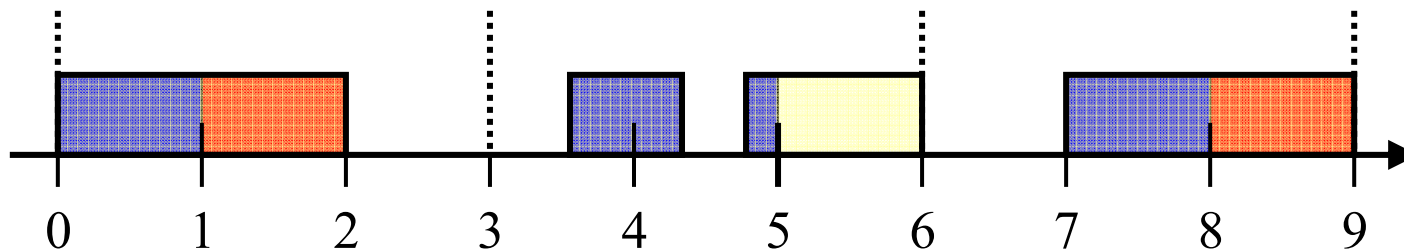
- Given a task group  $G$  such that
  - Scheduling algorithm : EDF
  - A set of periodic tasks :  $\{ T_1(3,1), T_2(7,1) \}$ , model the timing requirements of the task group with a periodic task model
- $G(3, 1.43)$  based on utilization does not work !!



# Simple Observation (2)

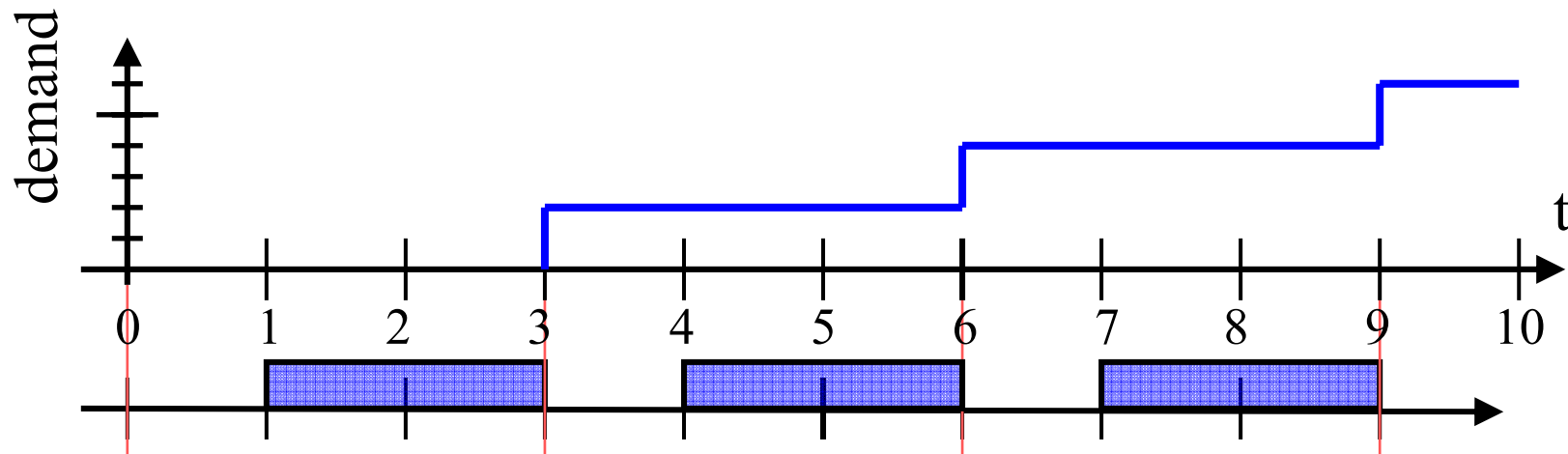
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- Given a task group  $G$  such that
  - Scheduling algorithm : EDF
  - A set of periodic tasks :  $\{ T_1(3,1), T_2(7,1) \}$ ,  
model the timing requirements of the task group with a periodic task model
- $G(3, 2.01)$  works !!



# Resource Demand Bound

- Resource demand bound during an interval of length  $t$ 
  - $dbf(W,A,t)$  computes the **maximum possible resource demand** that  $W$  requires under algorithm  $A$  during a time interval of length  $t$
- Periodic task model  $T(p,e)$  [Liu & Layland, '73]
  - characterizes the periodic behavior of resource demand with period  $p$  and execution time  $e$
  - Ex:  $T(3,2)$



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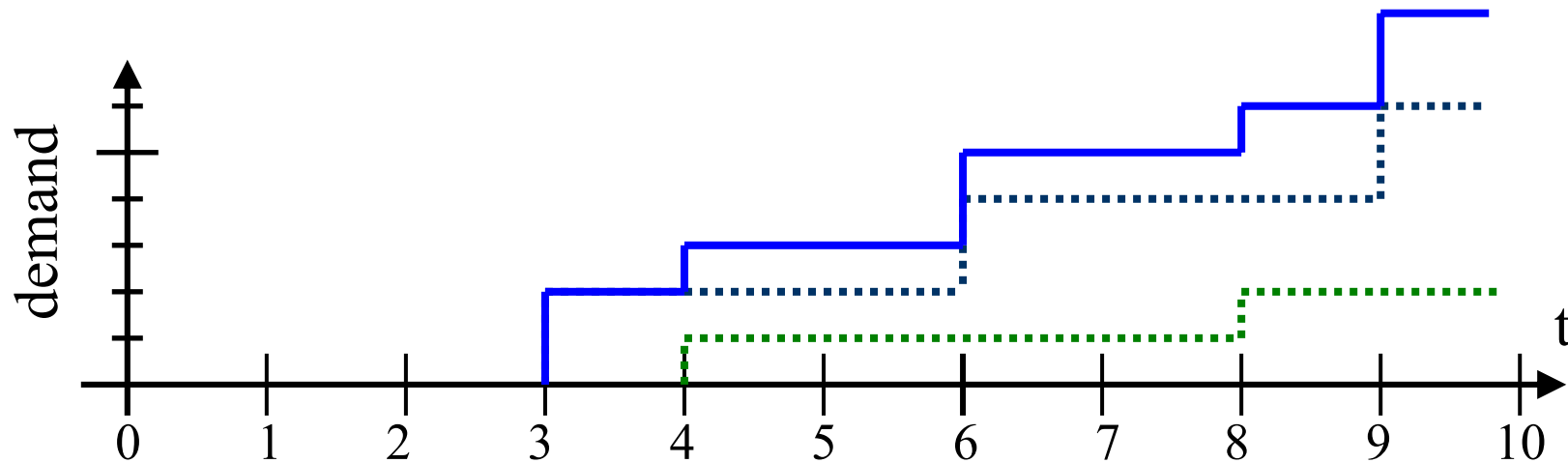
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# Demand Bound - EDF

- For a periodic workload set  $W = \{T_i(p_i, e_i)\}$ ,
  - $dbf(W, A, t)$  for EDF algorithm [Baruah et al., '90]

$$dbf(W, EDF, t) = \sum_{T_i \in W} \left\lfloor \frac{t}{p_i} \right\rfloor \cdot e_i$$





# ACSR

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$$T_1 = \{(cpu, 1)\} : \emptyset : T_1$$

$$T_2 = \{(cpu, 1)\} : \emptyset : T_2$$

$$+\emptyset : \{(cpu, 1)\} : \emptyset : T_2$$

$$T_3 = \{(cpu, 1)\} : \emptyset : \emptyset : T_3$$

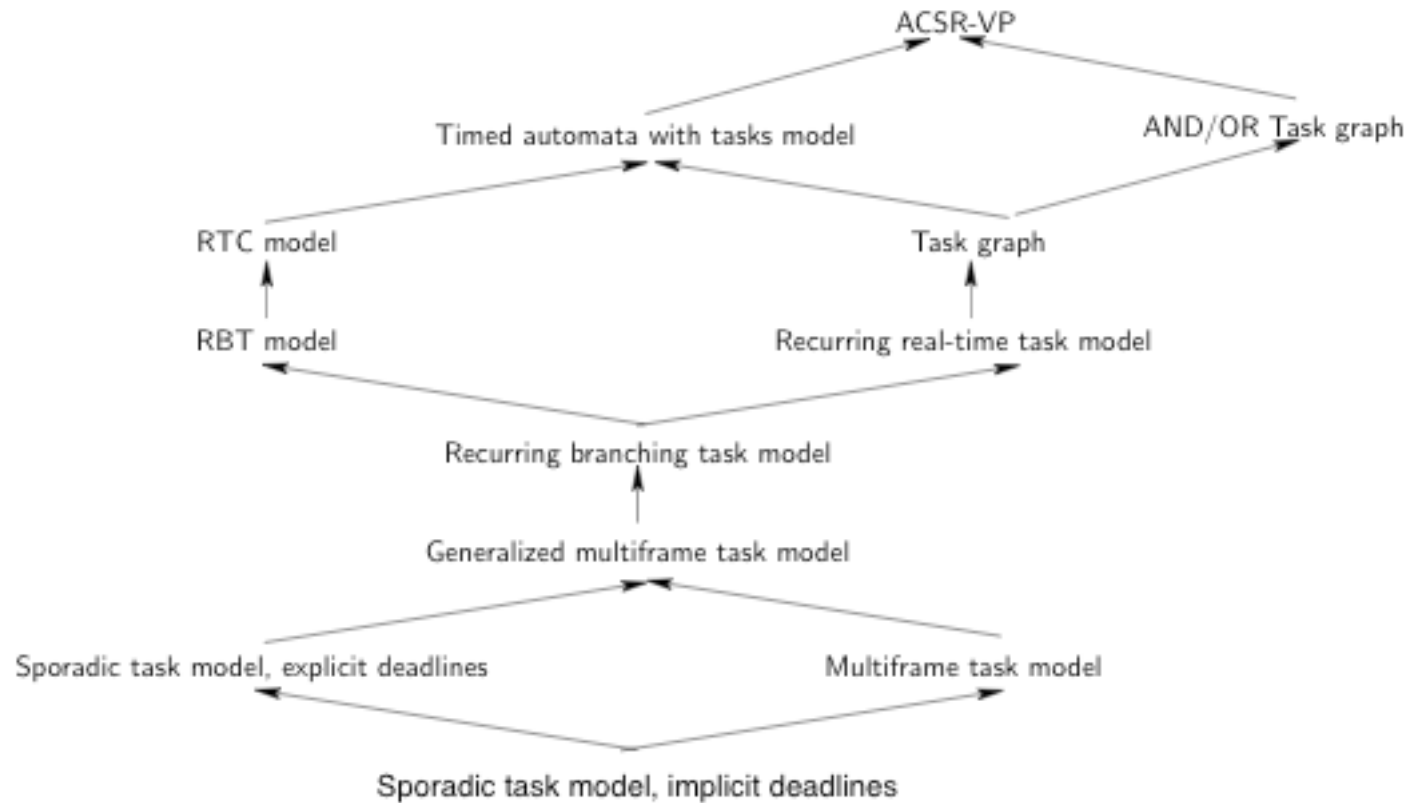
$$+\emptyset : \{(cpu, 1)\} : \emptyset : T_3$$

$$+\emptyset : \emptyset : \{(cpu, 1)\} : T_3$$



# Task (resource demand) representations

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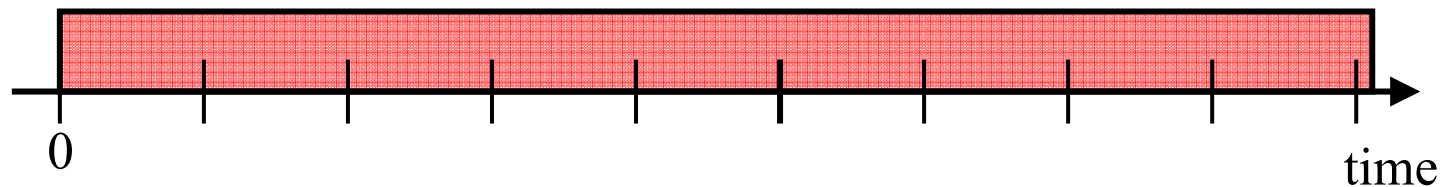


# Resource Supply Models

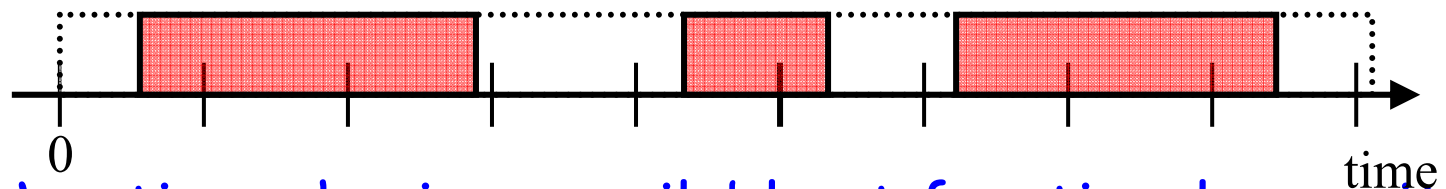
# Resource Modeling

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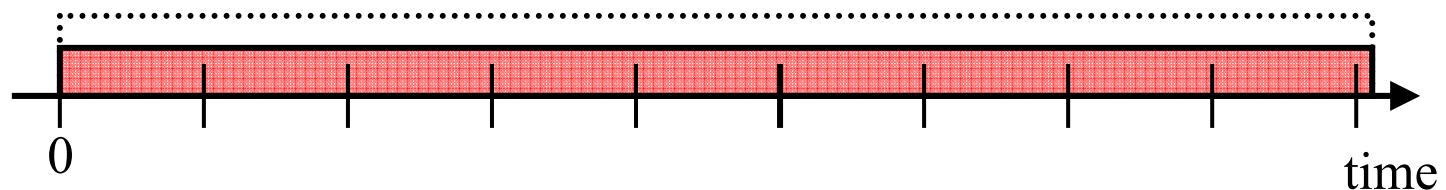
- Dedicated resource : always available at full capacity



- Shared resource : not a dedicated resource
  - Time-sharing : available at some times



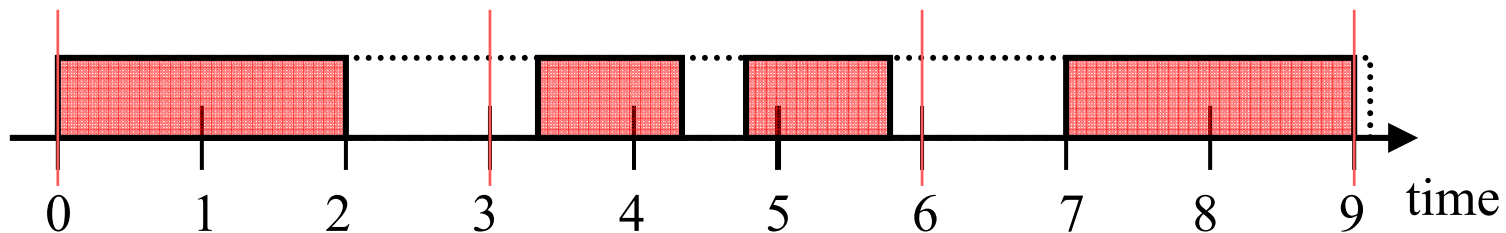
- Non-time-sharing : available at fractional capacity



# Resource Modeling

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- Time-sharing resources
  - Bounded-delay resource model [Mok et al., '01] characterizes a time-sharing resource w.r.t. a non-time-sharing resource
  - Periodic resource model  $\Gamma(\Pi, \Theta)$  [Shin & Lee, RTSS '03] characterizes periodic resource allocations
  - EDP model [Easwaran et al., RTSS 07]



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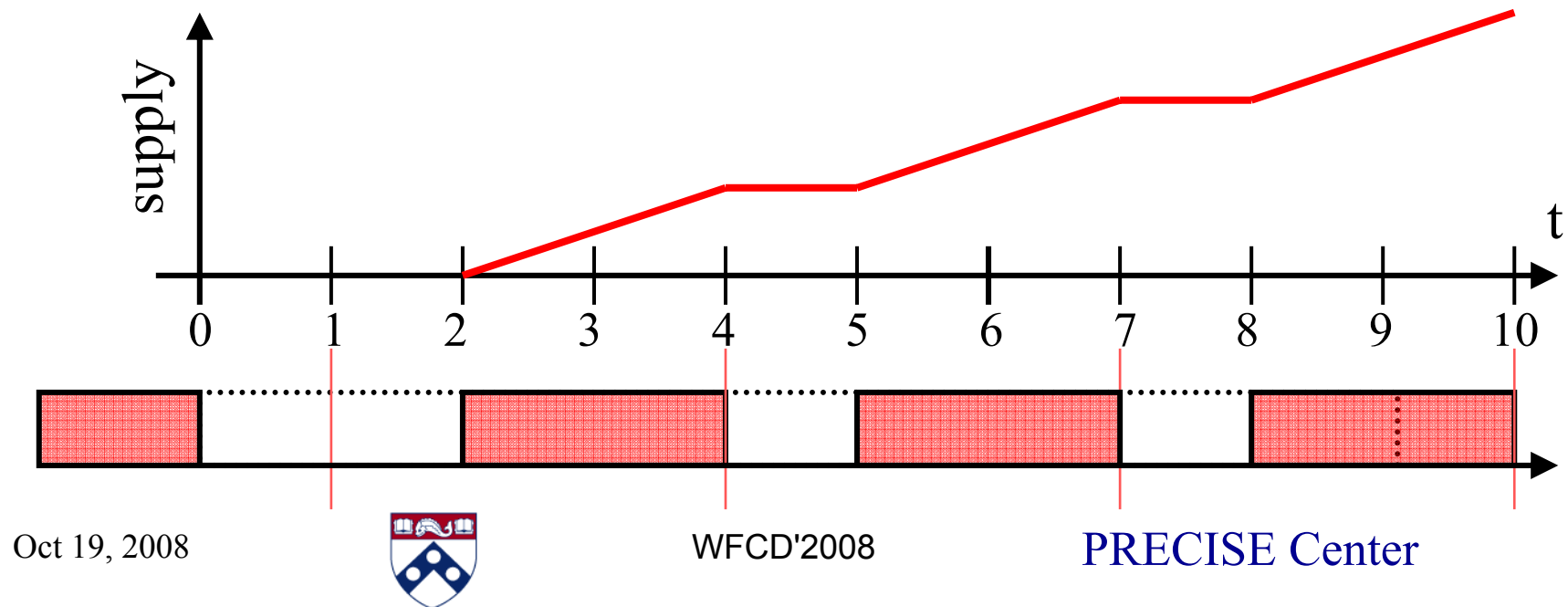


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# Resource Supply Bound

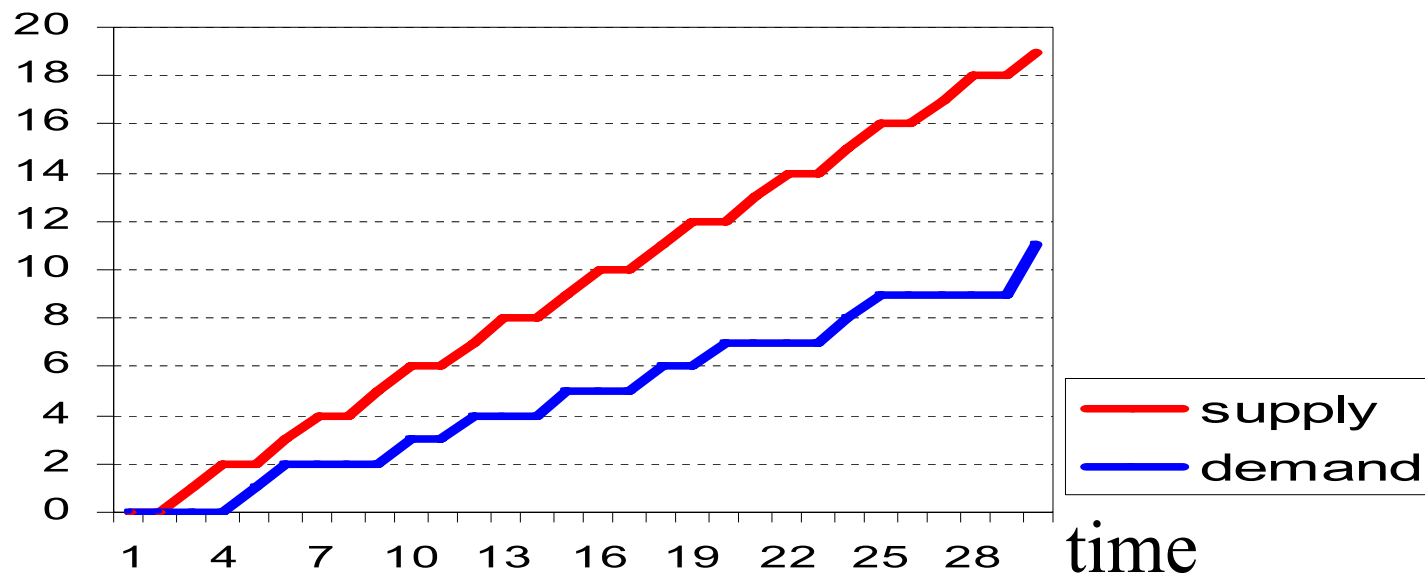
- Resource supply during an interval of length  $t$ 
  - $sbf_R(t)$ : the minimum possible resource supply by resource  $R$  over all intervals of length  $t$
- For a single periodic resource model, i.e.,  $\Gamma(3,2)$ 
  - we can identify the worst-case resource allocation



# Schedulability Condition - EDF

- A periodic workload set  $W$  is schedulable under a scheduling algorithm  $A$  over a periodic resource model  $\Gamma(\Pi, \Theta)$  if and only if  $\forall t > 0 \quad \text{dbf}(W, \text{EDF}, t) \leq \text{sbf}_\Gamma(t)$

-  $A = \text{EDF}$



# EDP resource model based Interfaces

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- Explicit Deadline Periodic resource
- Specification:  $\Omega = (\Pi, \Theta, \Delta)$ 
  - Explicit deadline  $\Delta$
  - $\Theta$  resource units in  $\Delta$  time units
  - Repeat supply every  $\Pi$  time units





# EDP supply bound function ( $sbf_{\Omega}$ )

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- $sbf_{\Omega}(t)$

$$sbf_{\Omega}(t) = y\Theta + \max\{0, t - (\Pi + \Delta - 2\Theta) - y\Pi\}$$

where  $y = \left\lfloor \frac{t - (\Delta - \Theta)}{\Pi} \right\rfloor, t \geq \Delta - \Theta$

- $lsbf_{\Omega}(t)$

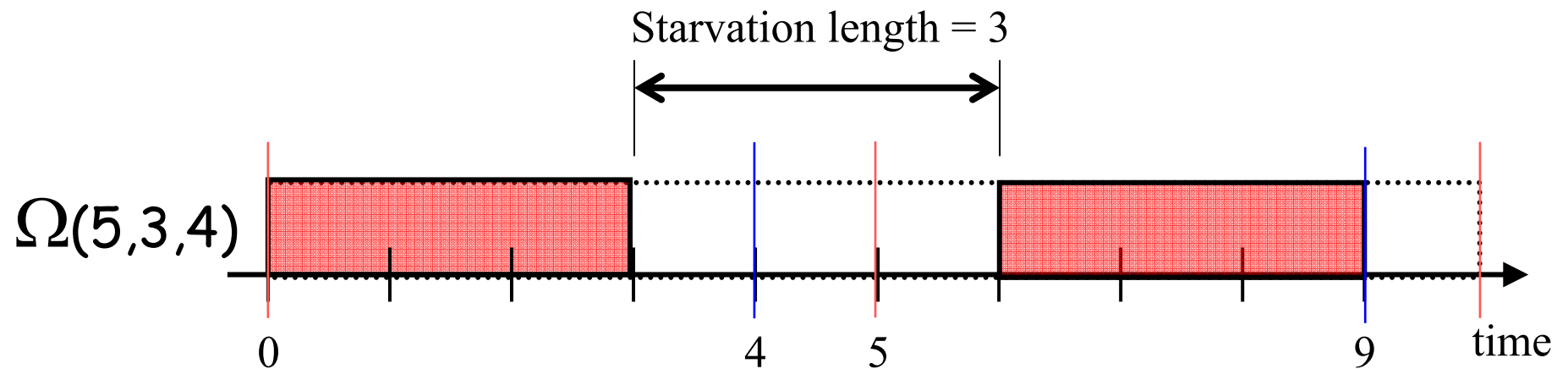
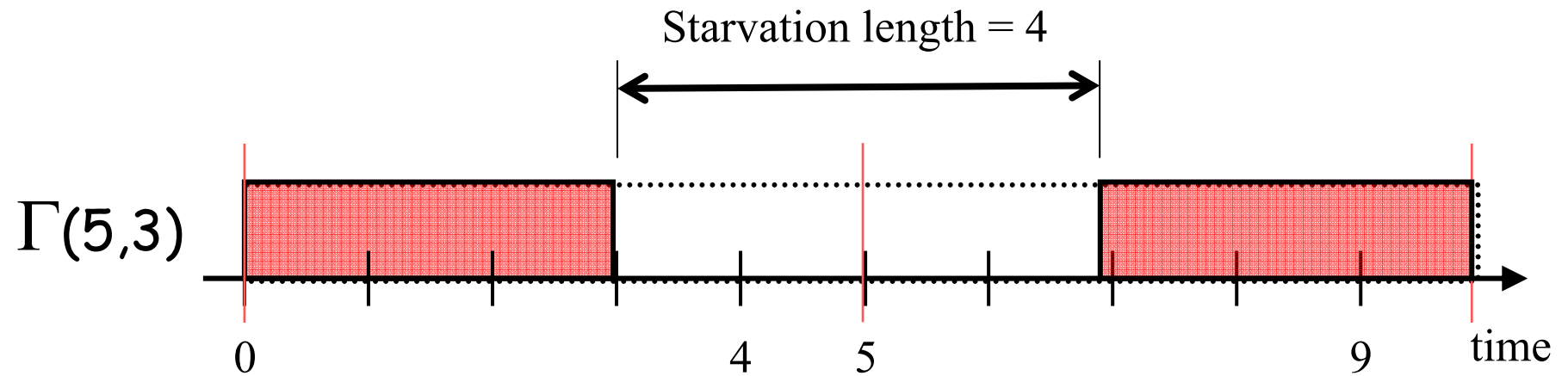
$$lsbf_{\Omega}(t) = \frac{\Theta}{\Pi} (t - (\Pi + \Delta - 2\Theta))$$

Bandwidth

Starvation length



# Supply bound function ( $\text{sbf}_{\Omega}$ )



# ACSR+ for supply partition specification

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$$S_1 = \{(cpu, -)\} : \emptyset : S_1$$

$$S_2 = \{(cpu, -)\} : \emptyset : S_2 \oplus \emptyset : \{(cpu, -)\} : S_2$$

$$T_1 = \{(cpu, 1)\} : \emptyset : T_1$$

$$T_2 = \{(cpu, 1)\} : \emptyset : T_2$$

$$+\emptyset : \{(cpu, 1)\} : \emptyset : T_2$$

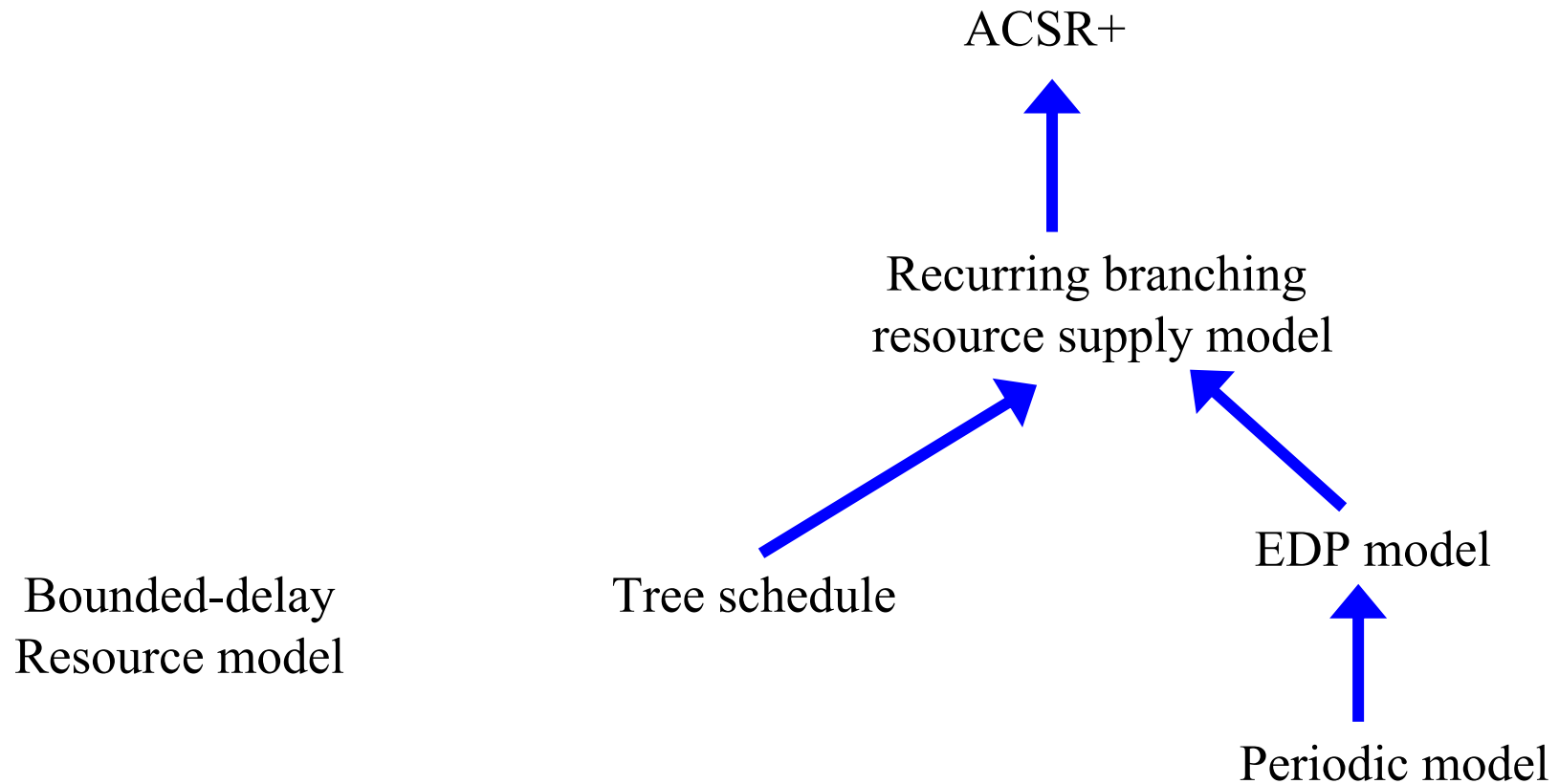
Notion of “schedulable under”

- (1)  $T_1$  is schedulable under  $S_1$
- (2)  $T_2$  is schedulable under  $S_2$
- (3)  $T_1$  is not schedulable under  $S_2$



# Resource Supply Models

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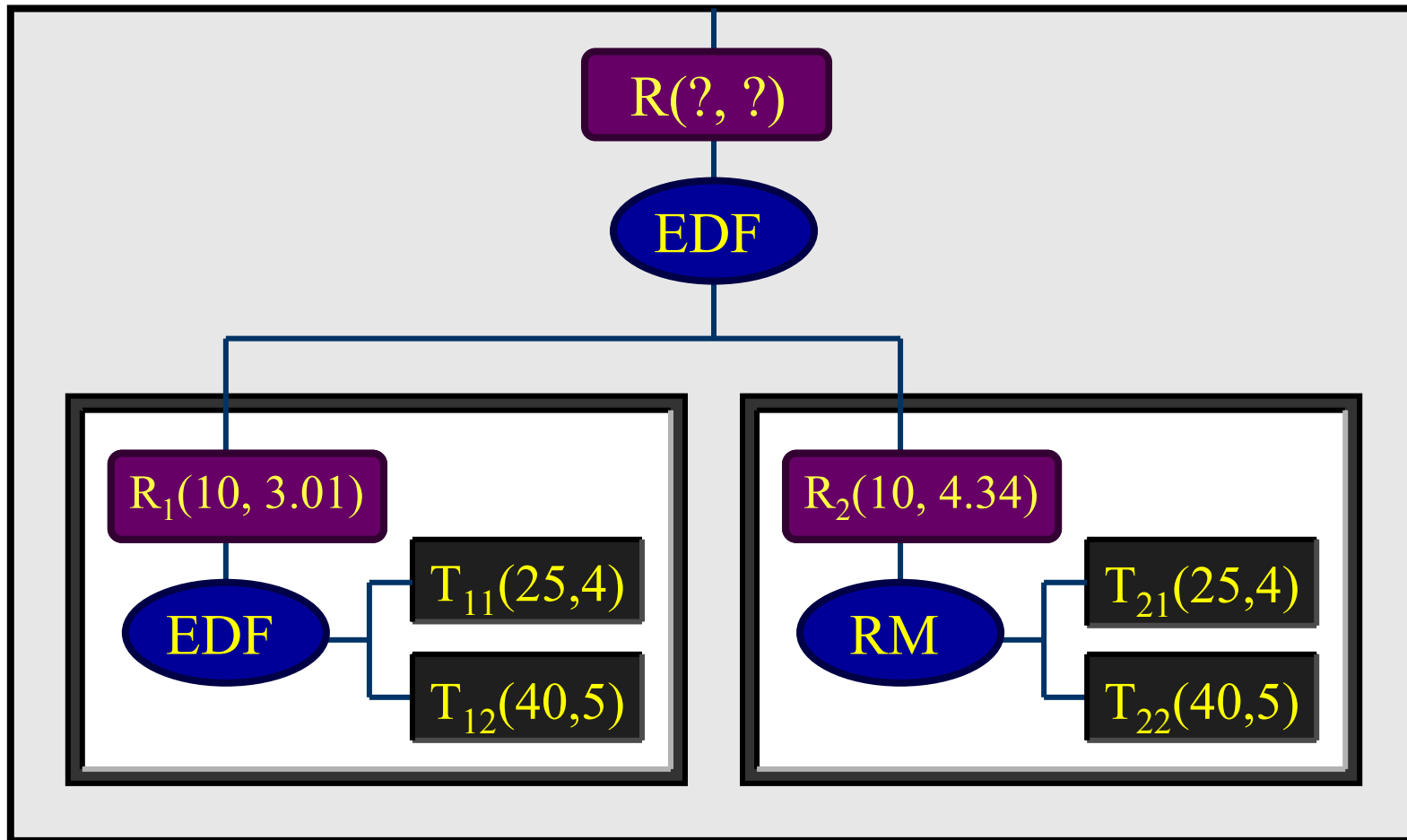




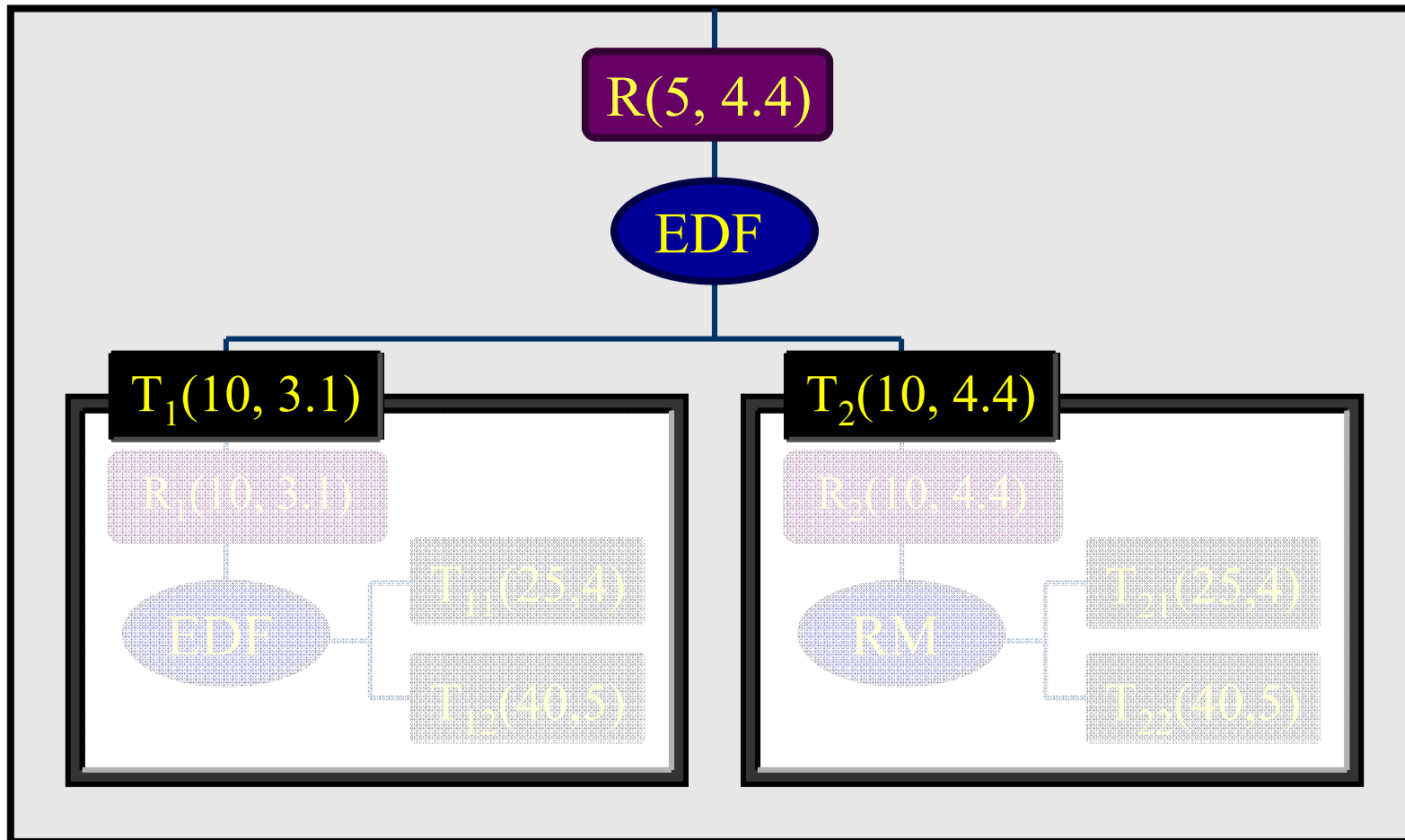
# Compositional Schedulability Analysis

# Component Abstraction

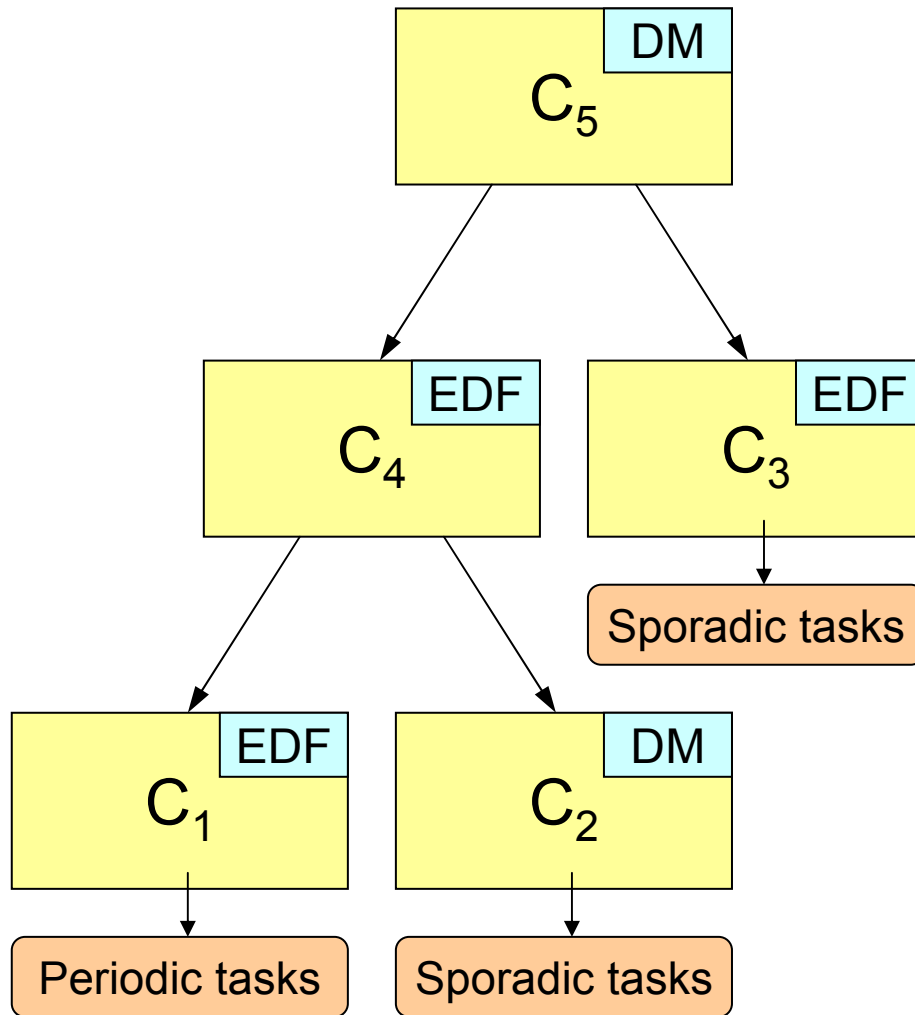
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# Compositional Real-Time Guarantees



# Hierarchical Scheduling Framework



- Resource allocation from parent to child
- Notations
  - Leaf  $\rightarrow C_1, C_2, C_3$
  - Non-leaf  $\rightarrow C_4, C_5$
  - Root  $\rightarrow C_5$

ARINC 653  $\rightarrow$  Two-level hierarchical framework





# Compositional Schedulability Analysis (CSA)

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- Assume/Guarantee reasoning
  - Let  $C\_R$  be a system configuration: Component  $C$  is running on resource  $R$ .
  - Let  $\text{supply}(C\_R)$  be the residual supply of  $R$  after  $C$ ; I.e., supply to the rest of the system.
  - If
    - $C1$  guarantees schedulability assuming  $\text{demand}(C1) \leq \text{supply}(C2\_R)$
    - $C2$  guarantees schedulability assuming  $\text{demand}(C2) \leq \text{supply}(C1\_R)$
  - Then,
    - $C1 \parallel C2$  guarantees schedulability in  $(C1 \parallel C2)\_R$



# Questions on CSA

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- Dbf/sbf bounds
  - Associativity
  - Minimum bounds on hierarchical scheduling
- ACSR/ACSR+
  - Non-deterministic supply alternatives
  - Definition and characterization of "schedulable under"
    - Given demand process  $T$  and supply partition  $S$ , when  $T$  schedulable with respect to  $S$ .
    - Relation to Linear Logic?



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# Hierarchical Scheduling Framework for Virtual Clustering of Multiprocessors

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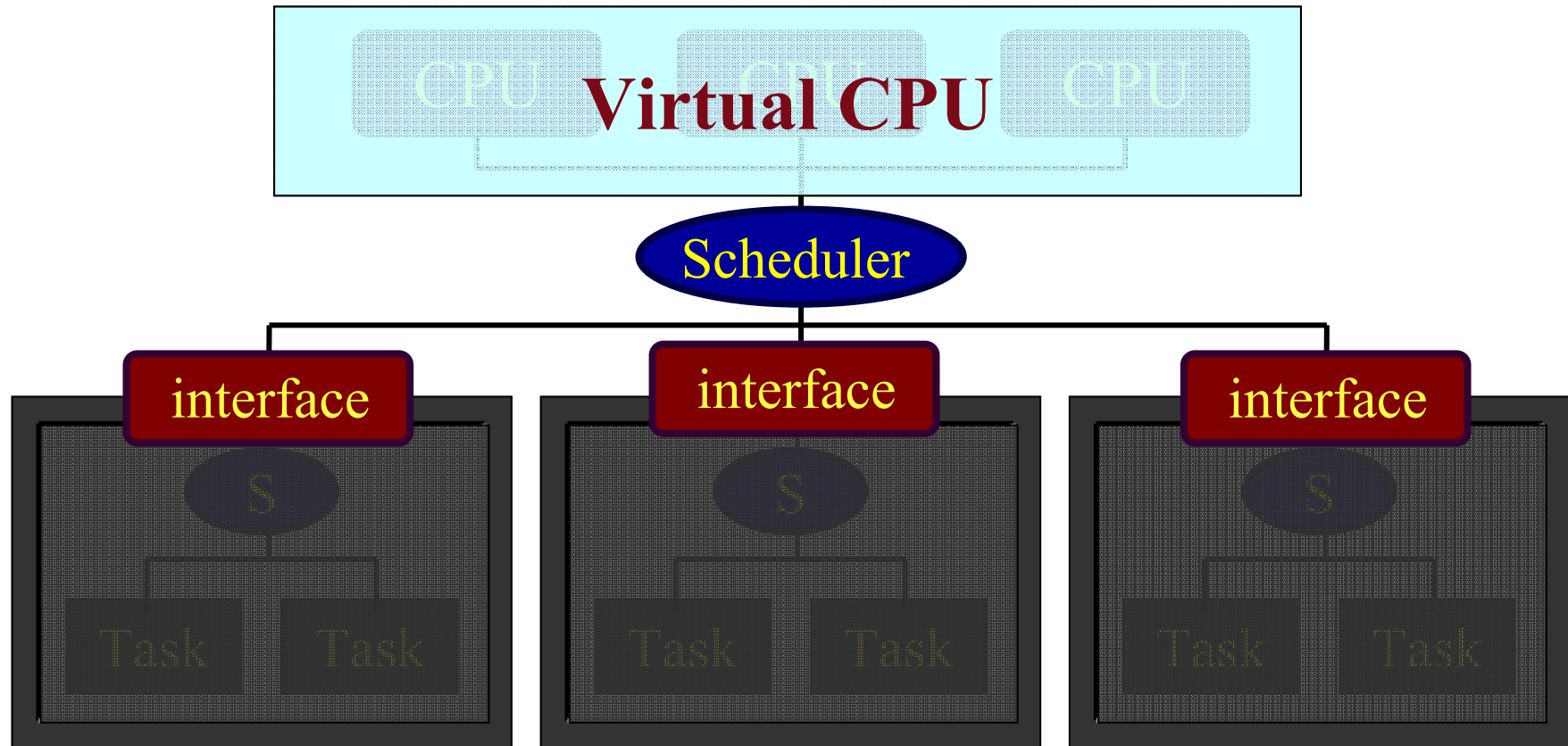


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# Multicore Processor Virtualization

1. Compositional analysis of hierarchical multiprocessor real-time systems, through component interfaces
2. Using virtualization to develop new component interface for multiprocessor platforms



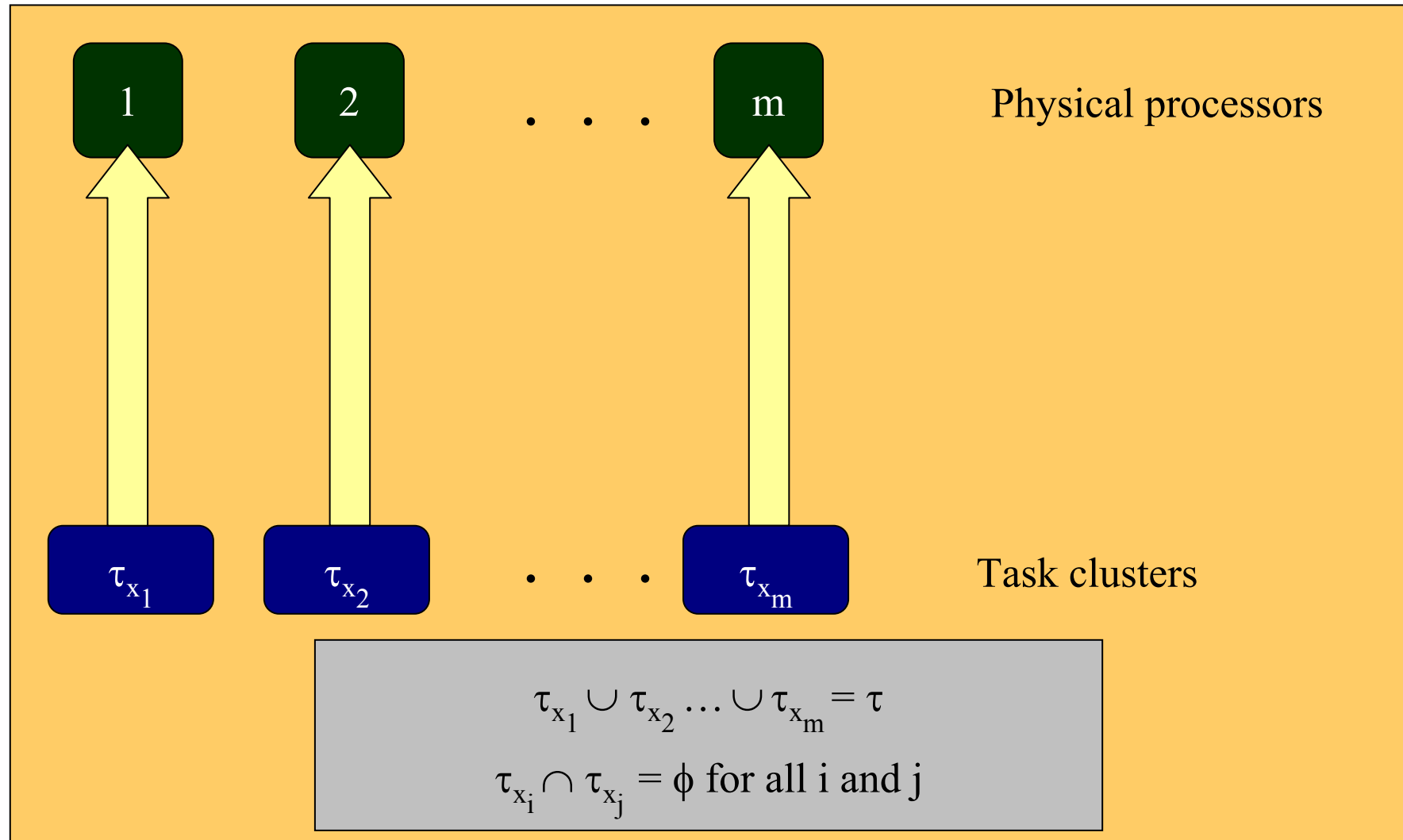
# Multiprocessor Embedded Systems

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- Why consider multiprocessors
  - Better tradeoff between computational power and costs (energy, fabrication)
  - Ability to exploit inherent concurrency in embedded software
- Problem Statement
  - Constrained deadline sporadic task model  
 $\tau = \{\tau_1, \dots, \tau_n\}$ , where each  $\tau_i = (T_i, C_i, D_i)$  with  $C_i \leq D_i \leq T_i$   
 $C_i$  units must be supplied non-concurrently
  - Identical, unit-capacity multiprocessor platform  
 $m$  processors
  - How can they be scheduled?

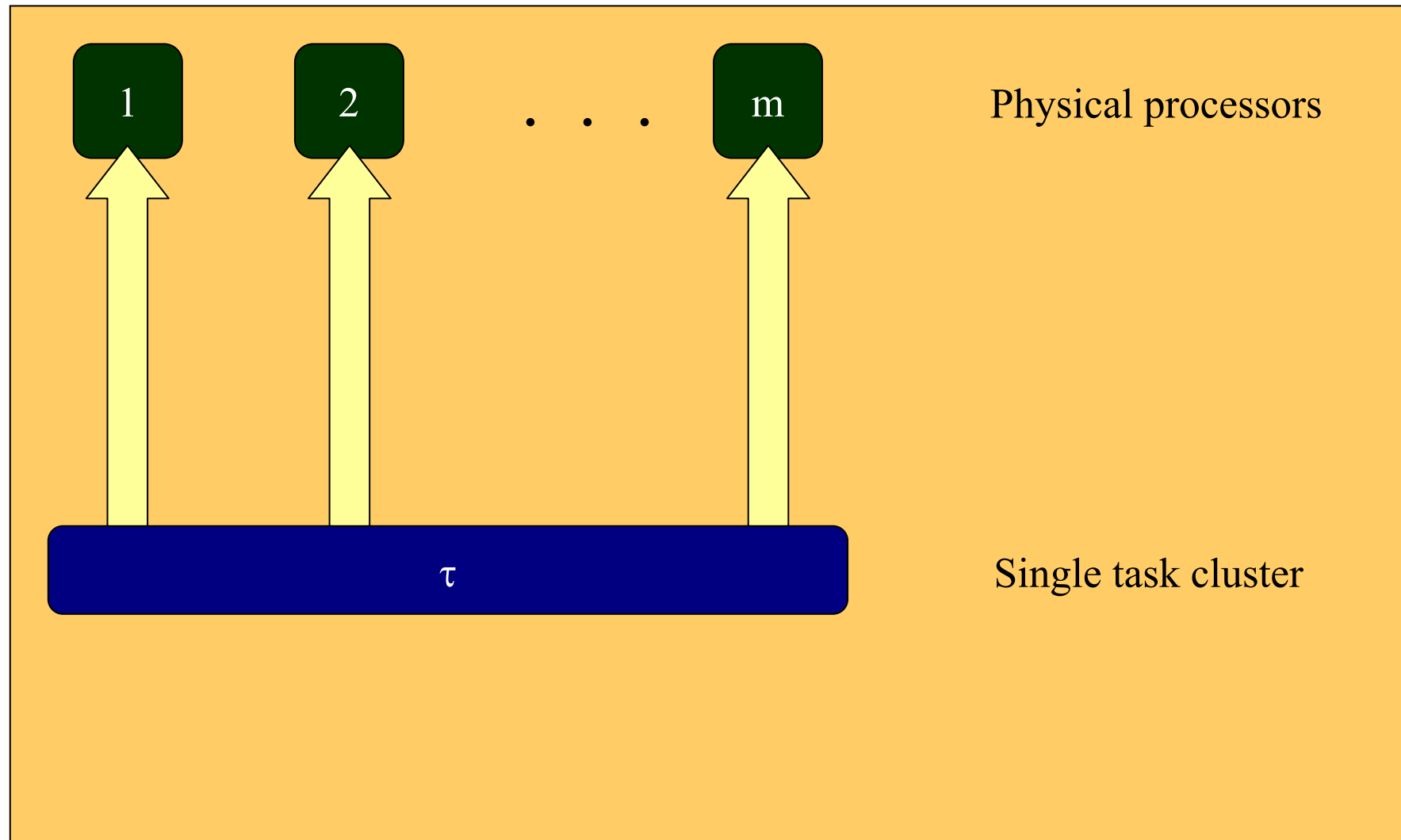


# Partitioned Scheduling



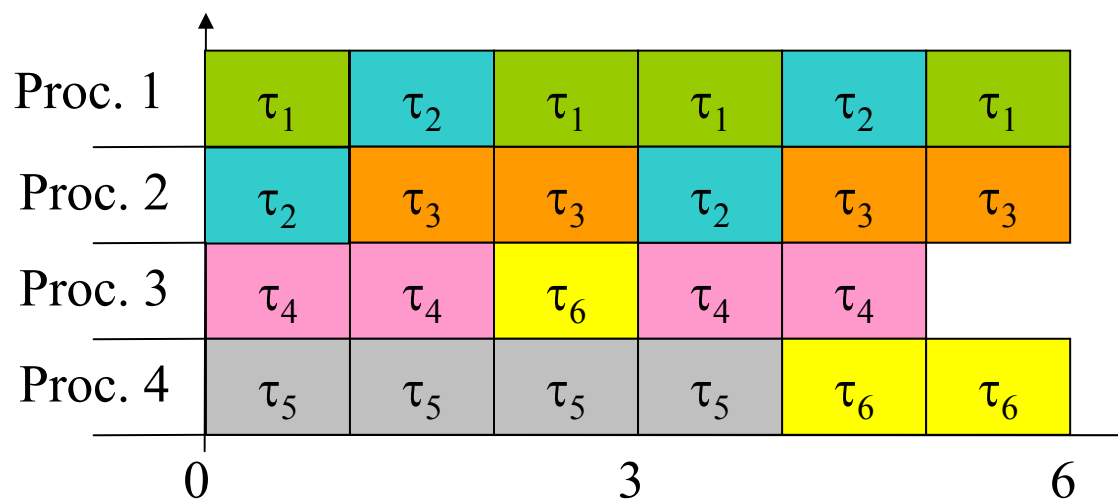
# Global Scheduling

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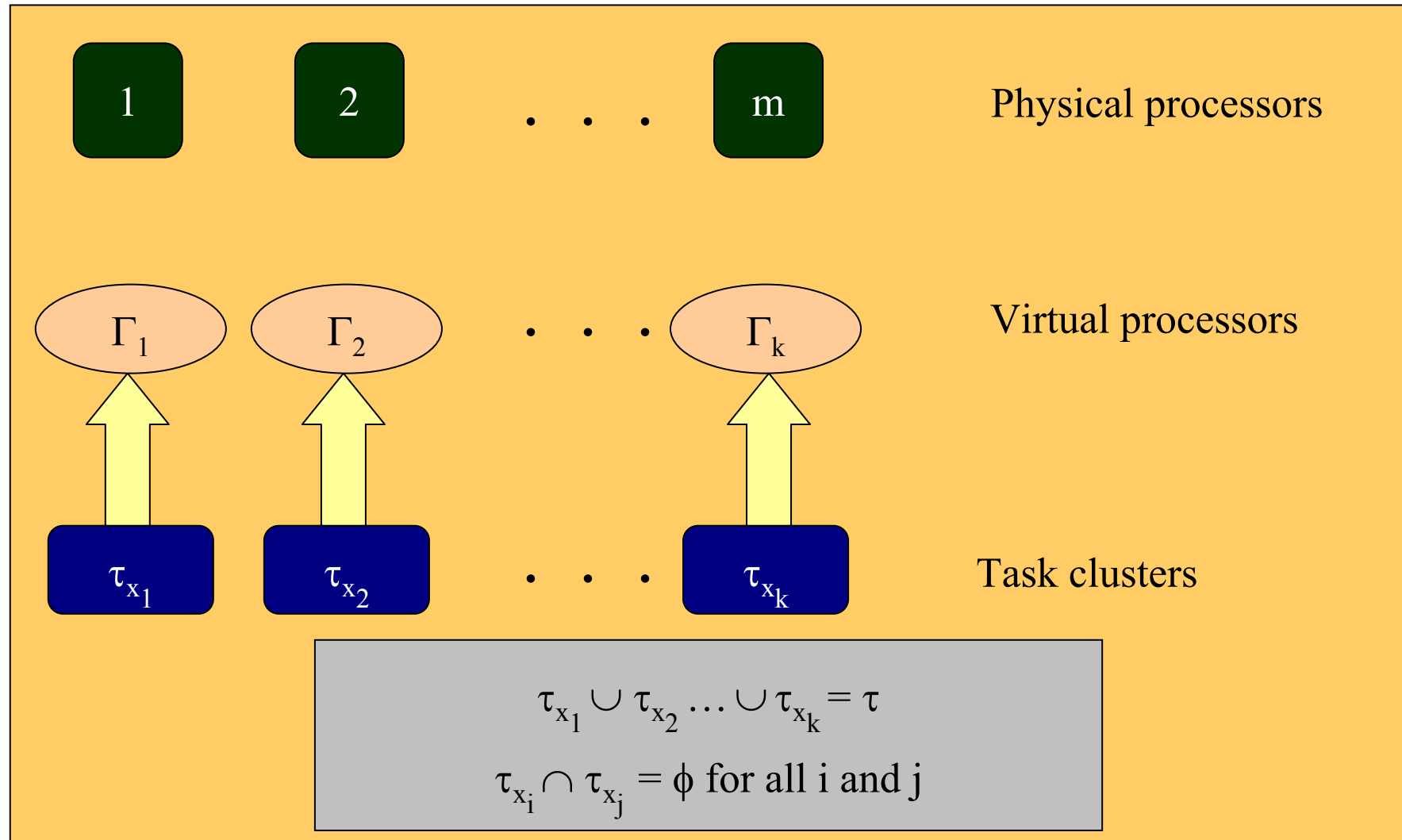
# Motivation for Virtual Clustering

- Task set and number of processors  
 $\tau_1=\tau_2=\tau_3=\tau_4=(3,2,3)$ ,  $\tau_5=(6,4,6)$ , and  $\tau_6=(6,3,6)$ ,  $m=4$
- Schedule under clustered scheduling  
 $\tau_1, \tau_2, \tau_3$  scheduled on processors 1 and 2  
 $\tau_4, \tau_5, \tau_6$  scheduled on processors 3 and 4

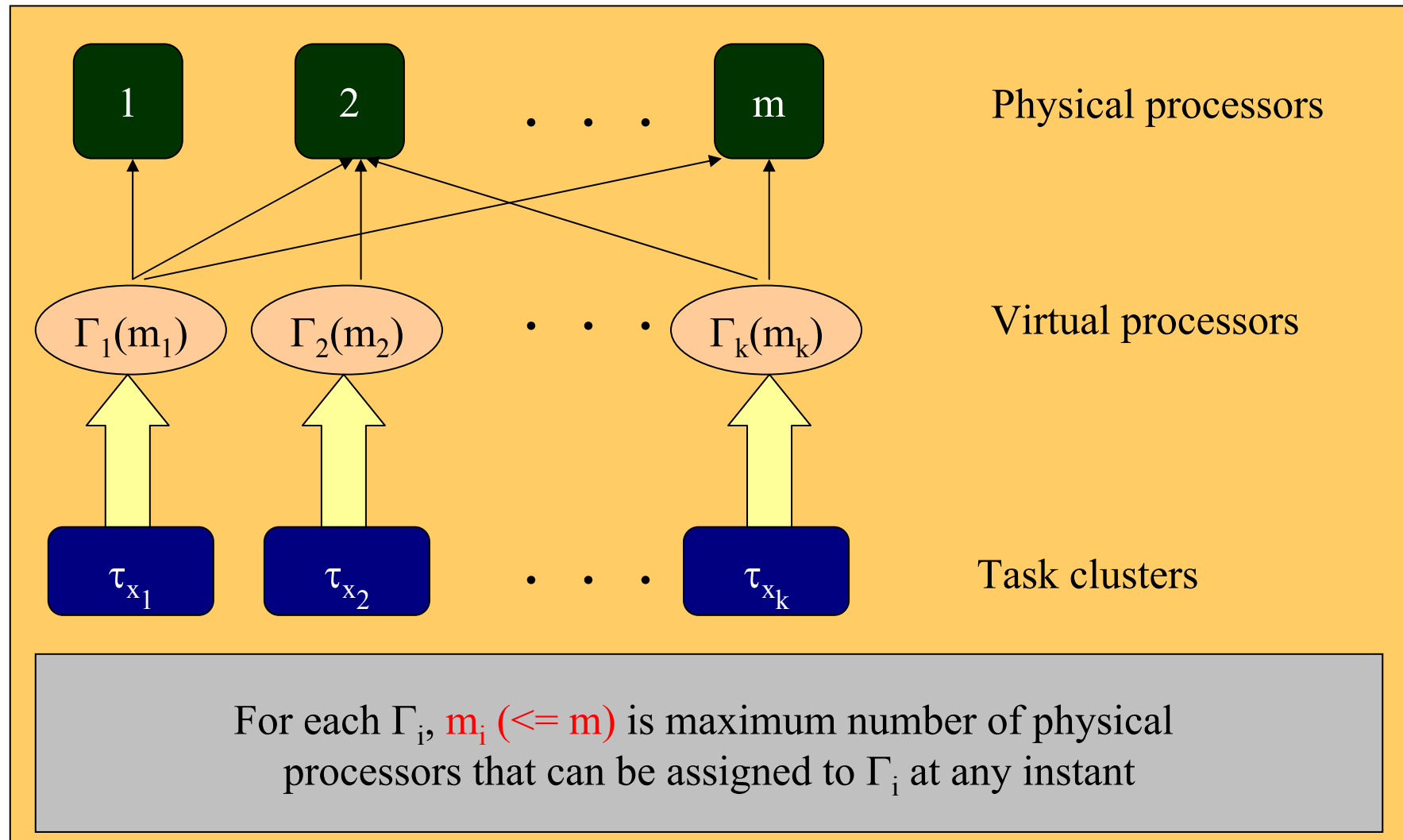




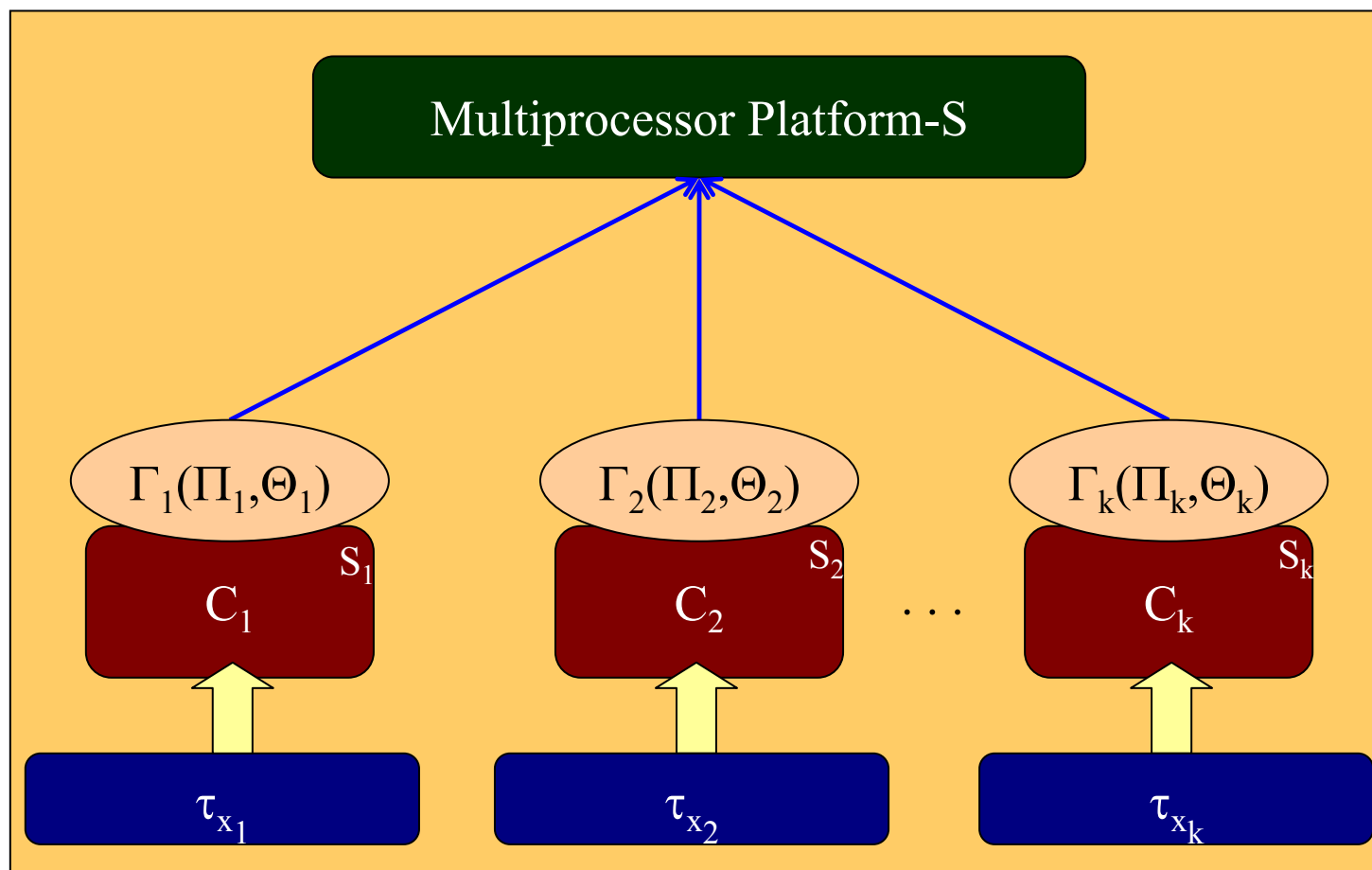
# Virtual Clustering Interface



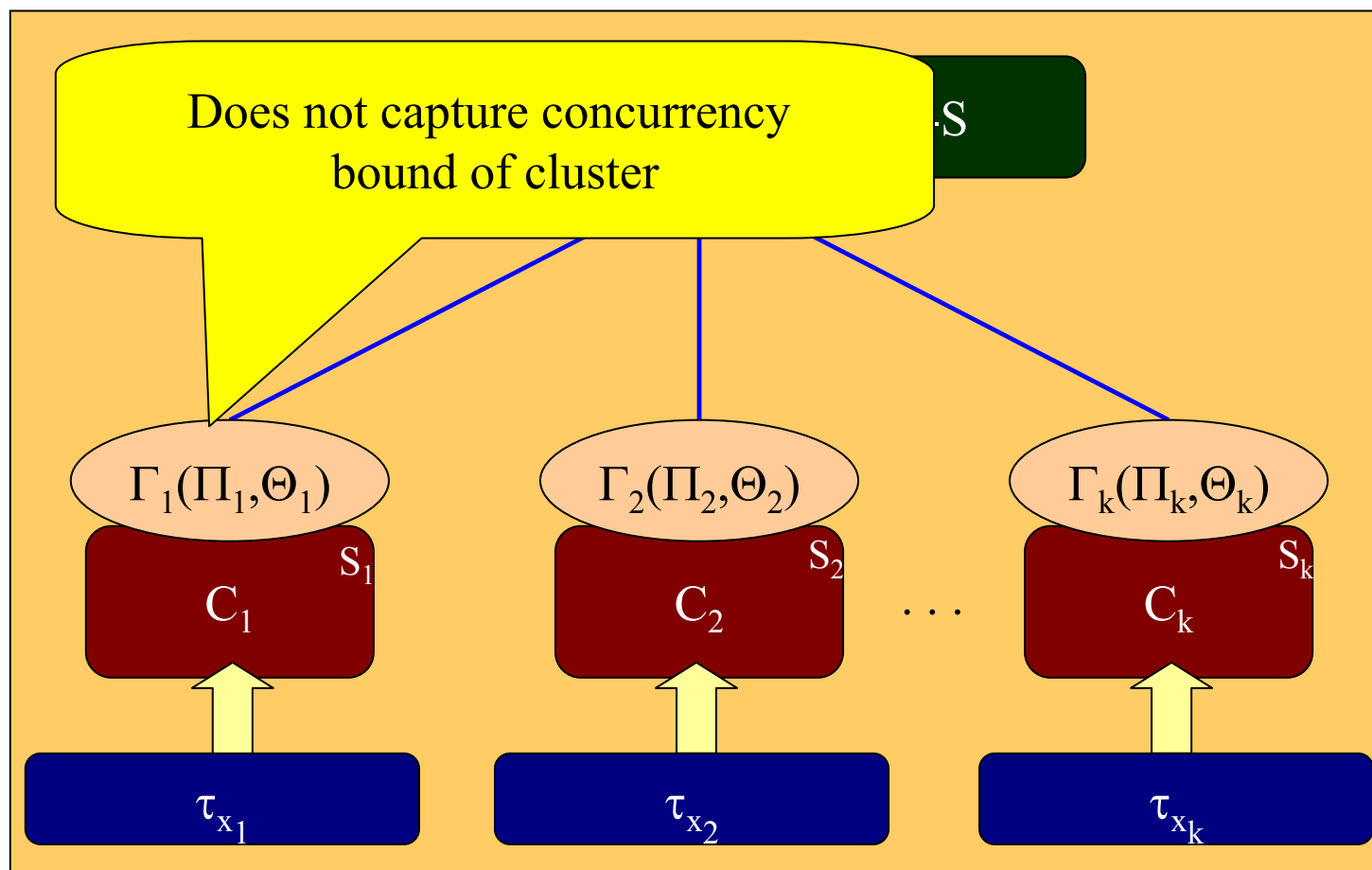
# Virtual Clustering Interface



# Multiprocessor Periodic Resource (MPR) model



# Multiprocessor Periodic Resource (MPR) model



# Multiprocessor Periodic Resource (MPR) model

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- $\Gamma = (\Pi, \Theta, m')$ 
  - $\Theta$  units of resource guaranteed in every  $\Pi$  units of time, with amount of concurrency at most  $m'$  in any time slot
- Why MPR model?
  - Periodicity enables transformation of resource model to tasks that can be used by various inter-cluster schedulers (schedulers at higher level)



# Virtual Cluster based Interface

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1. Split task set  $\tau$  into clusters  $\tau_{x_1}, \dots, \tau_{x_k}$ 
  - We assume that clusters are given
2. Abstract cluster  $\tau_{x_i}$  into MPR model  $\Gamma_i$ 
  - Solution for global EDF intra-cluster interface
  - Present sufficient schedulability condition and minimize overhead of  $\Gamma_i$
3. Transform each  $\Gamma_i$  into periodic tasks
  - Enables inter-cluster scheduler to schedule  $\Gamma_i$
  - Preserves concurrency bound of  $\Gamma_i$



# Conclusions

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- Interface framework for real-time system based on hierarchical schedulability analysis
  - Independent implementation of components
  - Interface-based component composition
  - Virtual clustering for multiprocessors
- Other issues
  - Task blocking due to synchronization
  - Context switch overheads
- Applications
  - ARINC 653
  - Automotive SAE J2056/Class C Vehicle Communication Requirements
  - Real-Time Virtual Machines (esp. multicore processors)



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- Incremental Schedulability Analysis of Hierarchical Real-Time Components, Easwaran et al., EMSOFT 2006.
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# Related work

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- Much work on hierarchical scheduling
  - Provide schedulability conditions that are needed for instantiation
  - Serves as the basis for abstraction
    - Shin and Lee, '03 '04, Easwaran et al., '06
- Real-time interface frameworks
  - Henzinger and Matic, '06
  - Wandeler and Thiele, '06



# Compositional Analysis (Dependency using Task Parameters)

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Questions?

Thank You!